Section 1.4

Definition of Function: A **function** f from a set A to a set B is a relation that assigns to each element x in the set A exactly one element y in the set B. The set A is the **domain** (or set of inputs) of the function f, and the set B contains the **range** (or set of outputs).

Characteristics of a Function from Set A to Set B

a) Each element in A must be matched with an element in B.

b) Some elements in B may not be matched with any element in A.

c) Two or more elements in A may be matched up with the same element in B.

d) An element in A (the domain) cannot be matched with different elements in B.

Function Terminology

Function: A function is a relationship between two variables such that to each value of the independent variable there corresponds exactly one value of the dependent variable.

Function Notation: $y = f(x)$; f is the name of the function, y is the **dependent variable**, x is the **independent variable**, and $f(x)$ is the value of the function at x.

Domain: The domain of a function is the set of all values (inputs) of the independent variable for which the function is defined. If x is in the domain of f, f is said to be defined at x. If x is not in the domain of f, f is said to be undefined at x.

Range: The range of a function is the set of all values (outputs) assumed by the dependent variable (that is, the set of all function values).

Implied Domain: If f is defined by an algebraic expression and the domain is not specified, the implied domain consists of all real numbers for which the expression is defined.

Problem 1. Let $A = \{a, b, c\}$, $B = \{0, 1, 2, 3\}$. Which sets of ordered pairs represent functions from A to B?

a) $\{(a, 1), (c, 2), (c, 3), (b, 3)\}$
b) $\{(a, 1), (b, 2), (c, 3)\}$
c) $\{(1, a), (0, a), (2, c), (3, b)\}$
d) $\{(c, 0), (b, 0), (a, 3)\}$

Problem 2. Determine whether the equation represents y as a function of x.

a) $x + y^2 = 4$

b) $(x+3)^2 + y^2 = 1$

c) $|y| = 4 - x$

d) $y = -5$

Problem 3. Evaluate the function at each specified value of the independent variable and simplify.

a) $h(t) = t^2 - 2t, \quad h(2), \quad h(-1), \quad h(x+2)$

b) $q(x) = \frac{1}{x^2-9}, \quad q(3), \quad q(y+3)$

c) $f(x) = \begin{cases} 2x+1, & x < 0 \\ 2x+2, & x \geq 0 \end{cases}, \quad f(2), \quad f(-2)$

d) $f(x) = \begin{cases} 2-3x, & x \leq -3 \\ 0, & -3 < x \leq 3 \\ 2x^2 - 8, & x > 3 \end{cases}, \quad f(-3), \quad f(-1), \quad f(4)$

Problem 4. In the following exercises, find the values of x for which $f(x) = g(x)$.

a) $f(x) = x^4 - 2x^2, \; g(x) = 2x^2$

b) $f(x) = \sqrt{x} - 4, \; g(x) = 2 - x$

Problem 5. Find the domain of the function.

a) $g(x) = 1 - 2x^2$

b) $f(t) = \sqrt[3]{t + 4}$

c) $h(x) = \frac{3}{x^2+3x+2}$

d) $f(x) = \frac{\sqrt{x+6}}{x+6}$

e) $f(x) = \frac{x-5}{\sqrt{x^2-9}}$

Problem 6. Find the difference quotient and simplify your answer.

$$f(x) = 5x - x^2, \qquad \frac{f(5+h) - f(5)}{h}, h \neq 0.$$

Homework: Read section 1.4, do #7, 9, 11, 12, 18, 19, 23-35 (odd), 39, 43, 45, 51, 53, 57, 59, 71 (the quiz for this section will be similar to these problems)